

Throughput and transmission capacity of underwater networks with randomly distributed nodes

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Abstract—In this paper, we derive fundamental performance limits of underwater (UW) networks via an analysis of the average behavior of random deployments. In particular, we consider an UW network that consists of a Poisson point process of transmitters, each with a receiver at a given distance. The link channel model accounts for path-loss, frequency-dependent absorption, and Rayleigh fading. We evaluate the probability of a successful transmission, defined as the probability that the signal-to-interference-and-noise ratio at the typical receiver is greater than a predetermined threshold, over different network and channel realizations. We then determine the network throughput density and the transmission capacity, defined as the maximum throughput density such that a constraint on the success probability is satisfied. The dependence of these metrics on the operating frequency and other system parameters is quantified through the proposed framework.

I. INTRODUCTION

Pioneering work in the field of UW networking [1] and later testbed deployments [2] have demonstrated that acoustic networks are feasible in practice. The main research thrusts in the field have so far targeted physical-layer and hardware design, channel characterization (see [3] for a recent review), as well as protocol design, comparison and evaluation [4]–[6]. Only a limited number of papers have focused on the analytical performance evaluation of UW networks and the derivation of fundamental limits. In particular, [7] highlighted the relationship between bandwidth and distance, and derived the capacity of an acoustic link. This study was extended in [8] to an UW cellular network setting with frequency reuse, and the tradeoff between the reuse factor and the maximum feasible user density was demonstrated. A different approach was taken in [9]: using asymptotic arguments, the authors showed that the amount of information that can be exchanged by each source-destination link in an UW network goes to zero roughly as $n^{-1/b}$, where n is the number of nodes and b is the path-loss exponent. In the same vein, [10] considered a multi-hop UW network with regular geometry, and showed that nearest-neighbor routing is order-optimal if the carrier frequency scales appropriately with the number of nodes.

In this paper, we take a different approach than in the aforementioned work, and focus on a random UW network with arbitrary node density. Our objective is to characterize the performance of a typical link of given parameters, e.g., transmit power, frequency and transmission distance, in the presence of both noise and interference from concurrent trans-

missions. In particular, we use a model that has been widely employed for the analysis of wireless radio networks, according to which the transmitters are assumed to be distributed in space according to a Poisson point process (PPP) (see [11] for a comprehensive overview and a list of references). Taking into account the characteristics of the UW channel, such as frequency-dependent absorption, fading and path-loss, we derive an exact expression and bounds for the probability of success of the typical link, defined as the probability that the received signal-to-noise-and-interference-ratio (SINR) is larger than a predetermined threshold, over different network and channel realizations. We then obtain the network throughput density and the transmission capacity, defined as the maximum throughput density such that a constraint on the success probability is satisfied. The main advantage of the proposed approach is that it yields explicit expressions for the metrics of interest as functions of the parameters of interest, in contrast to asymptotic analyses, which do not provide any information on the preconstants of the scaling laws [9], [10]. Moreover, it does not rely on any particular assumptions with respect to medium access control (MAC); the only relevant parameters are related to the acoustic communication and the capture model.

Through numerical results obtained for typical values of the operating frequency and the transmission power, it is demonstrated that the throughput-maximizing frequency is 2 to 4 times larger (depending on the transmission distance) than the frequency which maximizes the link signal-to-noise-ratio (SNR), providing up to a 7-fold improvement in the respective throughput. This is due to the frequency-dependent absorption of the interfering signals, which allows for a denser packing of transmissions, and is in agreement with the performance analysis of UW networks with frequency reuse in cellular-like topologies [8]. It is important to note here that packing more transmissions can be achieved in a physical sense, i.e., by deploying more nodes in a given area, but also by allowing the nodes in an existing deployment to transmit more frequently. In this sense, the results obtained in this paper can act as guidelines in determining MAC protocol parameters in practical UW networks.

II. SYSTEM MODEL AND METRIC DEFINITION

We consider an UW network that consists of an infinite number of transmitters (TX), each with a receiver (RX) at distance R and random orientation. The TXs are distributed

on the plane ($d = 2$ dimensions) or in space ($d = 3$) according to a PPP $\Phi = \{x\}$ of density λ , where x denotes the location of the typical TX. The power of each TX is P and is considered as constant. The channel power gain between any TX and RX at distance r and frequency f consists of (a) path-loss $r^{-b}a(f)^{-r}$, where, typically, $b \in [1, 2]$, and $a(f) > 1$ is the absorption factor, and (b) fading $h(f)$, where $h(f)$ is assumed to be exponentially distributed with unit mean (or $\sqrt{h(f)}$ is Rayleigh distributed¹). The fading random variables are i.i.d. across different TX-RX pairs. Moreover, we assume the presence of noise, which is additive with a power spectral density $W(f)$. For generally accepted empirical functions $W(f)$ and $a(f)$, the reader is referred to [7] and the references therein. In the remainder of the paper, we consider narrowband transmission, i.e., transmission that takes place in a “small” bandwidth δf around the carrier frequency f_o . Within δf , we assume that $W(f) = W(f_o)$, $a(f) = a(f_o)$, and $h(f)$ is constant and random². In the following, the dependence of W and a on f_o is implied.

Consider the RX corresponding to the typical TX located at x_0 , and assume (without loss of generality, due to the structure of the PPP) that it is placed at the origin. For given realizations of the PPP and the fading variables, the signal-to-interference-and-noise-ratio seen at the RX is

$$\text{SINR} = \frac{h_{x_0} R^{-b} a^{-R}}{W \delta f / P + I}, \quad (1)$$

where

$$I = \sum_{x \in \Phi \setminus \{x_0\}} h_x \|x\|^{-b} a^{-\|x\|}, \quad (2)$$

and h_x denotes the fading coefficient between the TX located at x and the RX, and $\|\cdot\|$ is the norm of x . We define the success probability, P_s , as the probability that the SINR satisfies a predetermined constraint θ , i.e.,

$$P_s = \mathbb{P}(\text{SINR} \geq \theta), \quad (3)$$

over all possible TX topologies and fading realizations. Taking an information-theoretic viewpoint, if $\text{SINR} \geq \theta$ is satisfied, and the noise, as well as the interference (given its power) are assumed to be Gaussian, then a rate of $\log_2(1 + \theta)$ (bits/symbol) is achievable. Similarly to the case of wireless radio networks, we define the following metrics [11]: the *throughput density* (or density of successful transmissions) $\tau(\lambda) = \lambda P_s(\lambda)$, which captures the tradeoff between the density of transmissions in space and the individual link quality; and the *transmission capacity* $c_\varepsilon = \lambda_\varepsilon(1 - \varepsilon)$, i.e., the maximum throughput density such that a constraint $P_s \geq 1 - \varepsilon$ is satisfied, where ε expresses the stringency of the SINR requirement, and depends on the application. Given the energy constraints of UW nodes, as well as the large delay penalties

¹The Rayleigh distribution models the medium-range shallow water channel accurately [12]; however, the proposed framework can also be extended to more general distributions.

²The case of wideband transmission and frequency selective channels can be handled by splitting the available bandwidth in sub-bands. The analysis presented here then applies analogously to each sub-band.

associated with retransmissions, imposing a constraint on the success probability and deriving the respective transmission capacity are particularly relevant to the operation of UW networks.

In the next section we obtain exact expressions and bounds for P_s , $\tau(\lambda)$ and c_ε .

III. EVALUATION OF THE SUCCESS PROBABILITY AND NETWORK METRICS

In order to evaluate the success probability defined in (3), we follow the approach outlined in [11] for the case of wireless radio networks. The main difference is the nature of the path-loss model of UW signal propagation, which results in a different expression for P_s .

A. Success probability

Since h_{x_0} is exponentially distributed, P_s can be written as [11, Eq. (9)]

$$P_s = \exp\left(-\frac{\theta R^b a^R W \delta f}{P}\right) \mathcal{L}_I(\theta R^b a^R) \triangleq P_{s,n} P_{s,i}, \quad (4)$$

where $\mathcal{L}_I(s)$, $s > 0$, is the Laplace transform of the probability density function of the interference, and $P_{s,n}$, $P_{s,i}$ denote the success probabilities taking into account only noise and interference, respectively. When $P \rightarrow \infty$ (and all other parameters are kept constant), the network is interference limited and $P_s = P_{s,i}$. Since Φ is a PPP, it is known that [13, Eq. (2.2), p. 292] (see also [11, Eq. (8)])

$$\mathcal{L}_I(s) = \exp\left(-\int_0^{+\infty} \mathbb{E}_h\left[1 - e^{-shr^{-b}a^{-r}}\right] \lambda_d(r) dr\right), \quad (5)$$

where $\lambda_d(r) \triangleq \lambda c_d dr^{d-1}$ and $c_d = \text{Vol}(B_d(0, 1))$ is the volume of the d -dimensional unit ball. For $d = 2$, we have $\lambda_2(r) = 2\lambda\pi r$, and for $d = 3$, we have $\lambda_3(r) = 4\lambda\pi r^2$. In the extreme case where $a = 1$, i.e., there is no absorption, $\mathcal{L}_I(s)$ is defined as long as $b > d$ [11]. This case corresponds to a wireless radio network and has been well studied in the literature [11]. In the following, we extend the analysis to $a > 1$, which is pertinent to the scenario of an UW network. Since $\mathbb{E}_h[e^{-sh}] = (1 + s)^{-1}$, from the definition of $P_{s,i}$ and (5), we obtain

$$P_{s,i} = \exp\left(-\lambda c_d d \theta R^b a^R \int_0^{+\infty} \frac{r^{d-1}}{r^b a^r + \theta R^b a^R} dr\right). \quad (6)$$

The integral in the exponent has no closed-form expression, but can be written as an infinite series, as shown in the following proposition. We recall the definitions of the lower incomplete Gamma function $\gamma(\zeta, x) = \int_0^x t^{\zeta-1} e^{-t} dt$, $x \in \mathbb{R}$, $\zeta > 0$, the upper incomplete Gamma function $\Gamma(\zeta, x) = \int_x^{+\infty} t^{\zeta-1} e^{-t} dt$, $x > 0$, $\zeta \in \mathbb{R}$, and the principal branch of the Lambert function $\mathcal{W}(x)$, $x \geq -e^{-1}$ [14], where $\mathcal{W}(x)$ is the unique solution of $ye^y = x$, $y \geq -1$.

Proposition 1 *The success probability in the interference-limited regime is*

$$P_{s,i} = e^{-\lambda c_d \sum_{n=0}^{\infty} A_n}, \quad (7)$$

where

$$A_0 = \bar{R}^d + \frac{d\theta R^b a^R}{(\log a)^{d-b}} \Gamma(d-b, \bar{R} \log a), \quad (8)$$

$$A_n = d\beta_n \gamma(d+bn, -n\bar{R} \log a) - d\beta_{-n-1} \Gamma(d-b(n+1), (n+1)\bar{R} \log a), \quad n \in \mathbb{Z}^+ \quad (9)$$

with

$$\beta_n = (-\theta R^b a^R)^{-n} (-n \log a)^{-d-bn} \quad (10)$$

and

$$\bar{R} = \frac{b}{\log a} \mathcal{W} \left(\frac{\log a}{b} (\theta R^b a^R)^{1/b} \right). \quad (11)$$

Proof: Denote the integral in (6) by \mathcal{I} . Then

$$\mathcal{I} = \int_0^{\bar{R}} \frac{(\theta R^b a^R)^{-1} r^{d-1}}{1 + \frac{r^b a^r}{\theta R^b a^R}} dr + \int_{\bar{R}}^{+\infty} \frac{(r^b a^r)^{-1} r^{d-1}}{1 + \frac{\theta R^b a^R}{r^b a^r}} dr, \quad (12)$$

where \bar{R} is such that $\bar{R}^b a^{\bar{R}} = \theta R^b a^R$ or $\bar{R} e^{\frac{\log a}{b} \bar{R}} = (\theta R^b a^R)^{1/b}$. Applying the Lambert function to both sides of this equation results in (11). Employing the series expansion $(1+x)^{-1} = \sum_{i=0}^{+\infty} (-x)^i$, $|x| < 1$, in (12), we obtain

$$\begin{aligned} \mathcal{I} &= \sum_{n=0}^{\infty} (-1)^n (\theta R^b a^R)^{-n-1} \int_0^{\bar{R}} r^{d+bn-1} a^{rn} dr \\ &+ \sum_{n=0}^{\infty} (-\theta R^b a^R)^n \int_{\bar{R}}^{+\infty} r^{d-b(n+1)-1} a^{-r(n+1)} dr. \end{aligned} \quad (13)$$

From (13) and the definitions of the incomplete Gamma functions, we obtain (7)-(10). \blacksquare

In the following corollary, we obtain arbitrarily tight bounds to $P_{s,i}$, that involve only a finite number of terms in the series in (7).

Corollary 1 *The success probability in the interference limited regime is bounded as*

$$e^{-\lambda c_d \sum_{n=0}^N A_n} < P_{s,i} < e^{-\lambda c_d \sum_{n=0}^{N-1} A_n}, \quad (14)$$

for any even integer $N > 0$. Moreover

$$e^{-\lambda c_d A_0} < P_{s,i} < e^{-\frac{\lambda c_d A_0}{2}}. \quad (15)$$

Proof: For $x \in (0, 1)$,

$$\frac{1}{1+x} = \sum_{n=0}^{N-1} (-x)^n + \sum_{n=N}^{+\infty} (-x)^n.$$

However,

$$\sum_{n=N}^{+\infty} (-x)^n = \frac{(-x)^N}{1+x} \leq 0,$$

which is > 0 for N even and < 0 for N odd. Therefore, for N even

$$\sum_{n=0}^{N-1} (-x)^n < \frac{1}{1+x} < \sum_{n=0}^N (-x)^n. \quad (16)$$

From (16) and (12), we obtain (14), following the same procedure as in the proof of Proposition 1. Eq. (15) is proved

similarly by employing the trivial bounds $1/2 < 1/(1+x) < 1$, for $x \in (0, 1)$. \blacksquare

Remarks on Proposition 1 and Corollary 1:

1. \bar{R} in (11) is the *critical radius*, defined as the distance at which the power from an interferer (averaged over the fading) is equal to $(\theta R^b a^r)^{-1}$. Alternatively, if we ignore fading, any interferer within a ball of this radius around the typical RX can cause an outage. By definition, if $\theta \geq 1$, then $\bar{R} \geq R$, and the equality holds for $\theta = 1$. If $\theta \rightarrow \infty$ (high-rate transmission) then $\bar{R} \rightarrow \infty$. With respect to the dependence of \bar{R} on the absorption factor, we have that, if $a \rightarrow 1$, then $\bar{R} \rightarrow \theta^{1/b} R$. For $a \rightarrow \infty$, with the application of de l'Hôpital's rule, (11) results in

$$\begin{aligned} \lim_{a \rightarrow \infty} \bar{R} &= \lim_{a \rightarrow \infty} \frac{b}{\log a} \mathcal{W} \left(\frac{\log a}{b} (\theta R^b a^R)^{1/b} \right) \\ &= b \lim_{a \rightarrow \infty} \frac{a^{R/b} (1 + R \log a/b)}{a^{R/b} \log a} = R, \end{aligned} \quad (17)$$

where the derivative of the Lambert function is obtained by differentiating $\mathcal{W}(x)e^{\mathcal{W}(x)} = x$ with respect to x .

2. Since the TX locations are obtained from a PPP, $P_{s,i}$ in (6) is equal to the probability that there are no TXs within a ball of volume

$$V_d = c_d \sum_{n=0}^{+\infty} A_n. \quad (18)$$

It can be verified that, for $\theta > 1$, V_d decreases as the absorption factor a increases. Since $a(f_o)$ is an increasing function of f_o [7], increasing the carrier frequency f_o improves the interference-limited success probability.

3. It is interesting to point out that (6) is defined for any positive value of b . This is not the case for a radio network with path-loss law r^{-b} , where the interference is almost surely infinite when $b \leq d$. In addition, employing Campbell's theorem [11], the mean interference power at the typical RX is

$$\mathbb{E}[I] = \lambda c_d d \int_0^{+\infty} r^{d-b-1} a^{-r} dr = \lambda c_d d \Gamma(d-b) (\log a)^{b-d}, \quad (19)$$

which is defined for $b < d$. Again, this is in contrast to radio networks with path-loss law r^{-b} , where the mean interference power is always infinite. The difference lies in the presence of the absorption factor $a > 1$, which ensures that the contribution of the far-away interferers goes to zero exponentially with distance. From Campbell's theorem, we also obtain that

$$\begin{aligned} \mathbb{E}[I_{\text{far}}] &= \lambda c_d d \int_{\bar{R}}^{+\infty} r^{d-b-1} a^{-r} dr \\ &= \lambda c_d d \Gamma(d-b, \bar{R} \log a) (\log a)^{b-d}, \end{aligned} \quad (20)$$

where I_{far} is the total interference power from transmitters located outside the ball of radius \bar{R} around the typical RX. Therefore, it becomes apparent that the second term in (8) is proportional to the ratio of the average interference power from "far" interferers to the average (over the fading) received signal power.

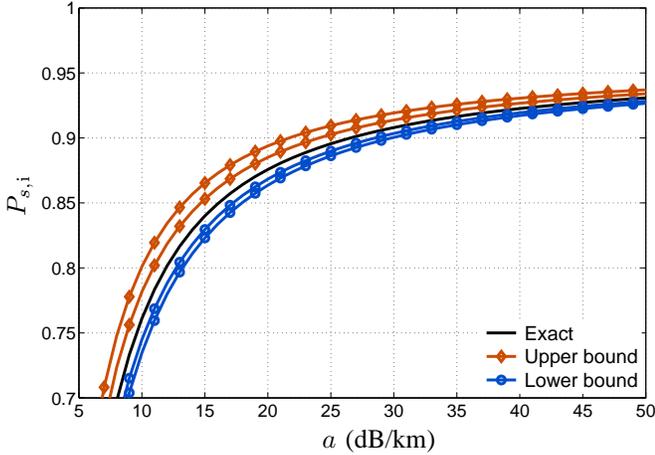


Figure 1. $P_{s,i}$ vs. a (dB/km) for $\lambda = 0.01$ nodes/km³ and $\theta = 10$ dB. The outer bounds are obtained by setting $N = 2$ in (14), and the inner bounds by setting $N = 4$. ($b = 1.5$, $d = 3$, $R = 1$ km.)

We close this section by showing that the lower bound in (15) is tight for $a \rightarrow \infty$ or $\theta \rightarrow \infty$.

Proposition 2 *If $\theta \rightarrow \infty$, then $P_{s,i} \sim e^{-\lambda c_d \bar{R}^d}$, where \sim denotes asymptotic equality. If $a \rightarrow \infty$, $P_{s,i} \rightarrow e^{-\lambda c_d R^d}$.*

Proof: Making use of the fact that $\Gamma(\zeta, x) \sim x^{\zeta-1} e^{-x}$ for $x \rightarrow \infty$, we can show that, for $a \rightarrow \infty$ or $\theta \rightarrow \infty$,

$$A_0 \sim \bar{R}^d \left(1 + \frac{d}{R \log a} \right), \quad (21)$$

and $\sum_{n=1}^{+\infty} A_n \rightarrow 0$. From (7), we therefore have that $P_{s,i} \sim e^{-\lambda c_d \bar{R}^d}$. In particular, if $a \rightarrow \infty$, then, from (17), $\bar{R} \rightarrow R$, which concludes the proof. ■

Proposition 2 implies that, in the limit of a large absorption factor, $P_{s,i}$ is equal to the probability that there are no interferers within a ball of radius R around the typical RX.

B. Network metrics

From the definition of $\tau(\lambda)$, (4), and Proposition 1, we have that

$$\tau(\lambda) = \lambda \exp \left(-\lambda V_d - \frac{\theta R^b a^R W \delta f}{P} \right). \quad (22)$$

Optimizing over the density of TXs λ , the optimal throughput density is

$$\tau_o = \frac{\exp \left(-\frac{\theta R^b a^R W \delta f}{P} - 1 \right)}{V_d}, \quad (23)$$

where V_d is as in (18). Similarly, from the definition of the transmission capacity, we obtain that

$$c_\varepsilon = \frac{-\log(1-\varepsilon) - \frac{\theta R^b a^R W \delta f}{P}}{V_d} (1-\varepsilon). \quad (24)$$

Note that c_ε is defined if $R^b a^R \leq -P \log(1-\varepsilon) / (\theta W \delta f)$. If we solve over R , we find that the maximum supportable transmission distance given ε is

$$R_{\max, \varepsilon} = \frac{b}{\log a} \mathcal{W} \left(\frac{\log a}{b} \left(-\frac{P \log(1-\varepsilon)}{\theta W \delta f} \right)^{1/b} \right). \quad (25)$$

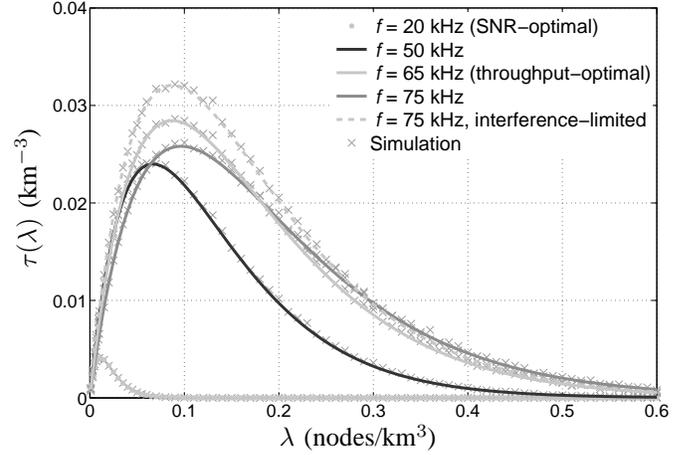


Figure 2. $\tau(\lambda)$ vs. λ for $R = 1$ km. The maximum throughput is achieved at $f = 65$ kHz. ($P/\delta f = 110$ dB re μ Pa/Hz, $b = 1.5$, $d = 3$.)

Beyond this value of R , the performance constraint cannot be satisfied and the transmission capacity of the network is zero.

IV. NUMERICAL RESULTS

Unless otherwise stated, we let $d = 3$, $b = 1.5$, $\theta = 10$ dB, $R = 1$ km, and $P/\delta f = 110$ dB re μ Pa/Hz. The absorption factor $a(f_o)$ and the noise power spectral density $W(f_o)$ are specified by (3) and (6) in [7], respectively.

We first consider the interference-limited case. In Fig. 1, $P_{s,i}$, evaluated from (6), is plotted as a function of a for a fixed TX density $\lambda = 0.01$ nodes/km³. The bounds obtained from (14) for $N = 2, 4$ are also plotted for comparison. It is seen that the bounds are quite tight even for such small values of N .

In Fig. 2, the throughput density $\tau(\lambda)$ is plotted as a function of λ for different operating frequencies at a transmission distance $R = 1$ km. The shape of the curves is typical of random-access (ALOHA) systems, i.e., for small λ , $\tau(\lambda)$ increases linearly in λ , while, for large λ , it decreases exponentially, and the maximum occurs at the density $\lambda_o = V_d^{-1}$. The curve for the interference-limited system shows the achievable throughput density when $P \rightarrow \infty$ for $f_o = 75$ kHz. The simulation results, obtained over 20000 network topologies with 10 different fading realizations per topology, confirm the validity of the analysis.

In Fig. 2, the maximum τ_o over all values of f_o considered is achieved at $f_o = 65$ kHz. The existence of a throughput-optimal frequency is apparent from (23): as f_o (or a) increases, the denominator decreases, i.e., the larger absorption factor allows for a denser packing of transmissions. At the same time, the nominator decreases, as the useful signal power also suffers from absorption³. In Fig. 3, τ_o and $c_{0.05}$, optimized over the operating frequency f_o , are plotted as functions of R . The respective optimal frequencies are shown in Fig. 4. For comparison, we also plot the frequency which maximizes

³The increase in f_o also results in a small decrease of the noise power $W(f_o)$ [7], but the absorption effect dominates.

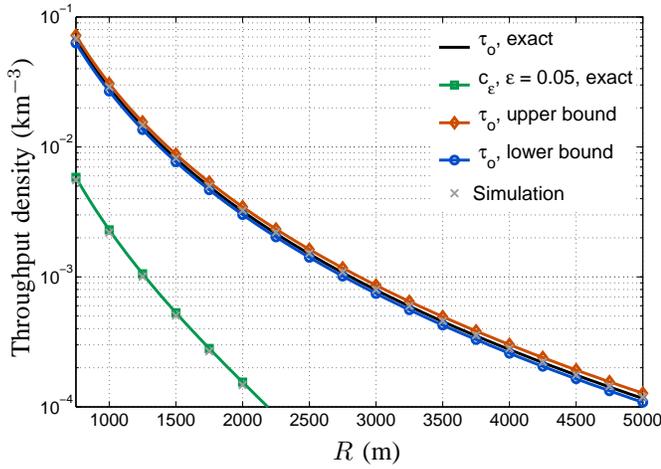


Figure 3. τ_o and $c_{0.05}$, optimized over f , are plotted vs. R . Imposing a constraint $1 - \varepsilon = 0.95$ on the success probability results in a less dense UW network and in a throughput loss. The bounds on the throughput are obtained by setting $N = 4$ in (14). ($P/\delta f = 110$ dB re $\mu\text{Pa}/\text{Hz}$, $b = 1.5$, $d = 3$.)

the received SNR $Pa(f_o)^{-R}R^{-b}/(W(f_o)\delta f)$ at each R . Note that the throughput-optimal frequency is significantly higher for moderate transmission distances, e.g., it is 65 kHz at $R = 1$ km, compared to the SNR-optimal value of 20 kHz. The respective throughput, as seen in Fig. 2, is about 7 times larger. As the transmission distance increases, the network becomes noise-limited and the gap between the different curves narrows. At $R \approx 3.5$ km, the frequency which maximizes $c_{0.05}$ is equal to the frequency which maximizes the SNR. Beyond this value of R , the constraint $1 - \varepsilon = 0.95$ on the success probability is no longer feasible (i.e., the numerator of (24) is negative for any value of f_o) and the transmission capacity of the UW network is zero.

V. CONCLUSIONS

We proposed an analytical framework to evaluate the throughput of UW networks, taking fully into account the specific propagation characteristics of the UW channel, as well as the dependence of the interference power on the TX locations. The framework is based on a random geometric approach, employed widely for the analysis of wireless radio networks, according to which the TX locations are drawn from a PPP. We obtained exact expressions and bounds to the success probability, the throughput density and the transmission capacity, as functions of salient parameters such as the transmission distance and the carrier frequency. A key observation from our numerical results is that, for moderate transmission distances, boosting the carrier frequency yields a significant throughput gain, since the benefit of the absorption of interfering signals outweighs the loss due to the absorption of the useful signal at the typical RX.

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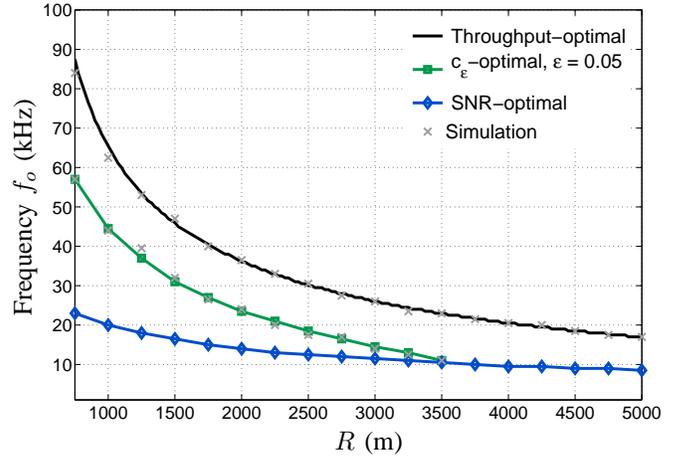


Figure 4. Optimal operating frequencies vs. R . Boosting the frequency for moderate transmission distances increases the throughput density, as it results in the absorption of interfering signals. ($P/\delta f = 110$ dB re $\mu\text{Pa}/\text{Hz}$, $b = 1.5$, $d = 3$.)

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